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A CALIBRATION METHOD FOR A SIX-PORT REFLECTOMETER WHICH MINIMIZES THE EFFECT OF POWER MEASUREMENT ERRORS

J.A. Dobrowolski, E. Bridges and L. Shafai

University of Manitoba, Department of Electrical Engineering

Winnipeg, Manitoba, Canada R3T 2N2

Summary

A calibration method for a six-port reflectometer which minimizes the effect of power measurement errors is presented. It is based on the minimization of an error expressions to obtain estimates of the six-port calibration constants. A load and four offset shorts are used as reflection standards. Computer simulation results are presented to show that the described procedure substantially reduces the overall error in the calibration constants.

Introduction

It is well known that the key problem for a six-port reflectometer is its calibration. The calibration of these devices has been studied extensively [1-6]. Calibration methods which have been described in the above mentioned papers do not take into account errors in power as obtained from the detector readings. However, any given set of power detector readings, $P_1, \ldots, P_4$, will differ from their true or correct values due to detector noise or other errors. This contribution attempts to remedy this situation. The calibration method described in this paper is based on the minimization of an error expression to obtain estimates of the calibration constants.

Computer simulation proves that the method developed may be effectively used for a single six-port reflectometer calibration.

Basic Equation of the Six-Port Reflectometer

For an arbitrary linear six-port microwave reflectometer, the basic relationship between its network parameters and the power readings can be expressed as follows:

$$p_i = \frac{P_i}{P_R} = Y_i \, \frac{1+2X_i|\Gamma_L|\cos(\phi_{xi}+\phi_L)+X_i^2|\Gamma_L|^2}{1+2Z|\Gamma_L|\cos(\phi_z+\phi_L)+Z^2|\Gamma_L|^2} \qquad (1)$$

where p_i, $i = 1,2,3$ are powers measured by three detectors attached to the six-port, P_R is the power measured by the reference detector and $|\Gamma_L|$ is the magnitude and ϕ_L the phase of the reflection coefficient to be tested [7]. The other quantities: Y_i, X_i, ϕ_{xi}; $i = 1,2,3$ and Z and ϕ_Z are real calibration constants. The purpose of a calibration procedure is to determine these quantities.

Calibration Method

A) Determination of the constants Y_i; $i = 1,2,3$.

An examination of Eq. (1) reveals that a matched load will suffice to determine Y_i; $i = 1,2,3$.

$$Y_i = p_i \; ; \; i = 1,2,3 \qquad (2)$$

The matched load is an absolute standard whose zero-magnitude reflection coefficent may be realized as the centroid of the locus of the reflection coefficient of an imperfect match as its phase is varied.

B) Determination of the constants Z and ϕ_z.

To determine the remaining calibration constants four unit magnitude offset standards with reflection phase angles ϕ_0, ϕ_1, ϕ_2 and ϕ_3 are used. From Eq. (1) one gets

$$\underline{A}\,\underline{z} + \underline{c} = [\underline{x}_1\underline{x}_2\underline{x}_3]^T\underline{b}_0 \qquad (3)$$

and

$$\underline{A}_i\,\underline{z} + \underline{c}_i = [\underline{b}_1\underline{b}_2\underline{b}_3]^T x_i \; ; \; i = 1,2,3, \qquad (4)$$

where

$$\underline{A} = \begin{bmatrix} R_{10} & 2R_{10}\cos\phi_0 & -2R_{10}\sin\phi_0 \\ R_{20} & 2R_{20}\cos\phi_0 & -2R_{20}\sin\phi_0 \\ R_{30} & 2R_{30}\cos\phi_0 & -2R_{30}\sin\phi_0 \end{bmatrix} \qquad (5)$$

$$\underline{A}_i = \begin{bmatrix} R_{i1} & 2R_{i1}\cos\phi_1 & -2R_{i1}\sin\phi_1 \\ R_{i2} & 2R_{i2}\cos\phi_2 & -2R_{i2}\sin\phi_2 \\ R_{i3} & 2R_{i3}\cos\phi_3 & -2R_{i3}\sin\phi_3 \end{bmatrix} ; \qquad (6)$$

$$\underline{b}_j = \begin{bmatrix} 1 \\ 2\cos\phi_j \\ -2\sin\phi_j \end{bmatrix}; \; \underline{c}_i = \begin{bmatrix} R_{i1} - 1 \\ R_{i2} - 1 \\ R_{i3} - 1 \end{bmatrix}; \; \underline{c} = \begin{bmatrix} R_{10} - 1 \\ R_{20} - 1 \\ R_{30} - 1 \end{bmatrix}; \qquad (7a)\,(7b)\,(7c)$$

$$\underline{z} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} Z^2 \\ Z\cos\phi_z \\ Z\sin\phi_z \end{bmatrix}; \; \underline{x}_i = \begin{bmatrix} x_{i1} \\ x_{i2} \\ x_{i3} \end{bmatrix} = \begin{bmatrix} X_i^2 \\ X_i\cos\phi_{xi} \\ X_i\sin\phi_{xi} \end{bmatrix}. \qquad (8a)\,(8b)$$

Here $R_{ij} = p_{ij}/Y_i$, $i = 1,2,3$ and $j = 0,1,2,3$ are the normalized powers measured by three power detectors ($i = 1,2,3$) for the four unit magnitude offset standards with reflection phase angles ϕ_j ($j = 0,1,2,3$). Eliminating $\underline{x}_i$, $i = 1,2,3$ from (3) and (4) yields

$$\underline{M}\,\underline{z} = \underline{d} \qquad (9)$$

where $\underline{M}$ is a 3x3 matrix whose elements are:

$M_{11} = R_{10} - (R_{11}Y_1 + R_{12}Y_2 + R_{13}Y_3)$

$M_{21} = R_{20} - (R_{21}Y_1 + R_{22}Y_2 + R_{23}Y_3)$

$$M_{31}=R_{30} - (R_{31}\gamma_1+R_{32}\gamma_2+R_{33}\gamma_3) \quad (10)$$

$$M_{12}=2(R_{10}\cos\phi_0-2R_{11}\gamma_1\cos\phi_1-2R_{12}\gamma_2\cos\phi_2-2R_{13}\gamma_3\cos\phi_3)$$

$$M_{22}=2(R_{20}\cos\phi_0-2R_{21}\gamma_1\cos\phi_1-2R_{22}\gamma_2\cos\phi_2-2R_{23}\gamma_3\cos\phi_3)$$

$$M_{32}=2(R_{30}\cos\phi_0-2R_{31}\gamma_1\cos\phi_1-2R_{32}\gamma_2\cos\phi_2-2R_{33}\gamma_3\cos\phi_3)$$

$$M_{13}=-2(R_{10}\sin\phi_0-2R_{11}\gamma_1\sin\phi_1-2R_{12}\gamma_2\cos\phi_2-2R_{13}\gamma_3\cos\phi_3)$$

$$M_{23}=-2(R_{20}\sin\phi_0-2R_{21}\gamma_1\sin\phi_1-2R_{22}\gamma_2\cos\phi_2-2R_{23}\gamma_3\cos\phi_3)$$

$$M_{33}=-2(R_{30}\sin\phi_0-2R_{31}\gamma_1\sin\phi_1-2R_{32}\gamma_2\cos\phi_2-2R_{33}\gamma_3\cos\phi_3)$$

with
$$\gamma_1 = \frac{1}{m}[\sin(\phi_2-\phi_3)+\sin(\phi_3-\phi_0)+\sin(\phi_0-\phi_2)]$$
$$\gamma_2 = \frac{1}{m}[\sin(\phi_3-\phi_1)+\sin(\phi_1-\phi_0)+\sin(\phi_0-\phi_3)]$$
$$\gamma_3 = \frac{1}{m}[\sin(\phi_1-\phi_2)+\sin(\phi_2-\phi_0)+\sin(\phi_0-\phi_1)]$$
$$m = \sin(\phi_1-\phi_3) + \sin(\phi_2-\phi_3) + \sin(\phi_3-\phi_1)$$
$$d_1 = -M_{11}$$
$$d_2 = -M_{21}$$
$$d_3 = -M_{31}$$

Any given set of power readings P_1, P_2, P_3 and P_R will differ from their correct values due to detector noise or other error. Thus it is not possible, in general, to choose values for Z and ϕ_z such that equation (9) are simultaneously satisfied. However, one may write

$$\underline{M}\,\underline{z} - \underline{d} = \underline{\varepsilon} \quad (11)$$

where $\underline{\varepsilon} = [\varepsilon_1, \varepsilon_2, \varepsilon_3]^T$.

An estimate of the Z and ϕ_z calibration constants can be obtained by choosing $z_2 = Z\cos\phi_z$ and $z_3 = Z\cos\phi_z$ to minimize a function

$$E = \underline{\varepsilon}^T\underline{\varepsilon} = \varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 \quad (12)$$

The solution to this is

$$\frac{\partial E}{\partial z_2} = 2\varepsilon_1 \frac{\partial\varepsilon_1}{\partial z_2} + 2\varepsilon_2 \frac{\partial\varepsilon_2}{\partial z_2} + 2\varepsilon_3 \frac{\partial\varepsilon_3}{\partial z_2} = 0 \quad (13a)$$

$$\frac{\partial E}{\partial z_3} = 2\varepsilon_1 \frac{\partial\varepsilon_1}{\partial z_3} + 2\varepsilon_2 \frac{\partial\varepsilon_2}{\partial z_3} + 2\varepsilon_3 \frac{\partial\varepsilon_3}{\partial z_3} = 0 \quad (13b)$$

According to (11) and because $z_1^2 = z_2^2 + z_3^2$, (13a) and (13b) create a set of two nonlinear equations with two unknowns $z_2 = Z\cos\phi_z$ and $z_3 = Z\sin\phi_z$. It may be solved using for example, the Newton iterative method [8].

The roots of Eqn.(13) may be computed iteratively using the formulas

$$z_2^{(k+1)} = z_2^{(k)} - \frac{1}{D_z}\left(Q_2 \frac{\partial Q_3}{\partial z_3}\bigg|_{z_2^{(k)}z_3^{(k)}} - Q_3\frac{\partial Q_3}{\partial z_3}\bigg|_{z_2^{(k)}z_3^{(k)}}\right)$$

$$z_3^{(k+1)} = z_3^{(k)} - \frac{1}{D_z}\left(Q_3 \frac{\partial Q_2}{\partial z_2}\bigg|_{z_2^{(k)},z_3^{(k)}} - Q_2 \frac{\partial Q_3}{\partial z_2}\bigg|_{z_2^{(k)}z_3^{(k)}}\right) \quad (14)$$

$$D_z = \left(\frac{\partial Q_2}{\partial z_2}\frac{\partial Q_3}{\partial z_3} - \frac{\partial Q_2}{\partial z_3}\frac{\partial Q_3}{\partial z_2}\right)\bigg|_{z_2^{(k)}z_3^{(k)}}$$

where k is the iteration index, and

$$Q_2 = \partial E/\partial z_2 = 2e(z_2^3+z_2z_3^2)+h(3z_2^2+z_3^2+2rz_2z_3 +(f+2e)z_2+tz_3+h \quad (15a)$$

$$Q_3 = \partial E/\partial z_3 = 2e(z_3^3+z_2^2z_3)+r(3z_3^2+z_2^2)+2hz_2z_3 +(g+2e)z_3+tz_2+r \quad (15b)$$

$$e = M_{11}^2 + M_{21}^2 + M_{31}^2 \; ; \quad f = M_{12}^2 + M_{22}^2 + M_{32}^2$$

$$g = M_{13}^2 + M_{23}^2 + M_{33}^2 \; ; \quad h = M_{11}M_{12}+M_{21}M_{22}+M_{31}M_{32}$$

$$r = M_{11}M_{13}+M_{21}M_{23}+M_{31}M_{33}; \; t = M_{12}M_{13}+M_{22}M_{23}+M_{32}M_{33}$$

Once z_2 and z_3 have been found, one has

$$Z = \sqrt{z_1^2 + z_2^2}, \text{ and } \phi_z = \arctan(z_3/z_2) \quad (16)$$

C) Determination of the constants X_i, ϕ_{xi}; i=1,2,3

Since Z and ϕ_z are now known, we can determine the rest of the calibration constants. Using (4) we get

$$\underline{B}\,\underline{x}_i = \underline{u}_i \; ; \; i = 1, 2, 3 \quad (17)$$

where
$$\underline{u}_i = \underline{A}_i\,\underline{z} + \underline{c}_i \quad (18)$$

and
$$\underline{B} = [\underline{b}_1\ \underline{b}_2\ \underline{b}_3]^T \quad (19)$$

Because of errors in the power readings, one may write (17) in the form

$$\underline{B}\,\underline{x}_i - \underline{u}_i = \underline{\mu}_i \; ; \; i = 1, 2, 3 \quad (20)$$

where $\underline{\mu}_i = [\mu_{i1}\ \mu_{i2}\ \mu_{i3}]^T$.

As in the case of Z and ϕ_z, an estimate of the X_i and ϕ_{xi} calibration constants can be obtained by choosing $x_{i2} = X_i\cos\phi_{xi}$ and $x_{i3} = X_i\sin\phi_{xi}$ to minimize the function

$$F_i = \mu_i^T \mu_i = \mu_{i1}^2 + \mu_{i2}^2 + \mu_{i3}^2 \quad (21)$$

The solution to this is

$$\frac{\partial F_i}{\partial x_{i2}} = 2\mu_{i1} \frac{\partial\mu_{i1}}{\partial x_{i2}} +2\mu_{i2} \frac{\partial\mu_{i2}}{\partial x_{i2}} +2\mu_{i3} \frac{\partial\mu_{i3}}{\partial x_{i2}} = 0 \quad (22a)$$

$$\frac{\partial F_i}{\partial x_{i3}} = 2\mu_{i1} \frac{\partial\mu_{i1}}{\partial x_{i3}} +2\mu_{i2} \frac{\partial\mu_{i2}}{\partial x_{i3}} +2\mu_{i3} \frac{\partial\mu_{i3}}{\partial x_{i3}} = 0 \quad (22b)$$

According to (20), and because $x_{i1}^2 = x_{i2}^2 + x_{i3}^2$, (22a) and (22b) create a set of two nonlinear equations with two unknowns $x_{i2} = X_i\cos\phi_{xi}$ and $x_{i3} = X_i\sin\phi_{xi}$. The roots of (22) may be computed iteratively the same way as it has been done for (13) using (14). This time we have

$$Q_2 = \partial F/\partial x_{i2} = 6x_{i2}^3+6x_{i2}x_{i3}^2+6\alpha x_{i2}^2+2\alpha x_{i3}^2-4\beta x_{i2}x_{i3} +2(2\gamma-\delta_i Z^2-2z_iZ\cos\phi_z+2\varepsilon_iZ\sin\phi_z-\delta_i+3)x_{i2} -4\zeta x_{i3}+2(\alpha-z_i-z_iZ^2+2\kappa_iZ\sin\phi_z-2\tau_iZ\cos\phi_z)$$

$$Q_3 = \partial F/\partial x_{i3} = 6x_{i3}^3+6x_{i2}^2x_{i3}-6\beta x_{i3}^2-2\beta x_{i2}^2+4\alpha x_{i2}x_{i3} +2(2\mu-\delta_i Z^2-2z_iZ\cos\phi_z+2\varepsilon_iZ\sin\phi_z-\delta_i+3)x_{i3} -4\zeta x_{i3}+2(\alpha-z_iZ^2+2\kappa_iZ\sin\phi_z-2\tau_iZ\cos\phi_z) \quad (22)$$

with
$$\alpha = \cos\phi_1 + \cos\phi_2 + \cos\phi_3$$
$$\beta = \sin\phi_1 + \sin\phi_2 + \sin\phi_3$$
$$\gamma = \cos^2\phi_1 + \cos^2\phi_2 + \cos^2\phi_3$$
$$\zeta = 0.5(\sin2\phi_1 + \sin2\phi_2 + \sin2\phi_3)$$
$$\mu = \sin^2\phi_1 + \sin^2\phi_2 + \sin^2\phi_3$$
$$\delta_i = R_{i1} + R_{i2} + R_{i3}$$
$$z_i = R_{i1}\cos\phi_1 + R_{i2}\cos\phi_2 + R_{i3}\cos\phi_3$$
$$\varepsilon_i = R_{i1}\sin\phi_1 + R_{i2}\sin\phi_2 + R_{i3}\sin\phi_3$$
$$\tau_i = R_{i1}\cos^2\phi_1 + R_{i2}\cos^2\phi_2 + R_{i3}\cos^2\phi_3$$
$$\kappa_i = 0.5(R_{i1}\sin2\phi_1 + R_{i2}\sin2\phi_2 + R_{i3}\sin2\phi_3)$$
$$\eta_i = R_{i1}\sin^2\phi_1 + R_{i2}\sin^2\phi_2 + R_{i3}\sin^2\phi_3$$

As an "initial estimate" for x_{i2} and x_{i3} to start the Newton iterative procedure to solve (22), one may use the solution of the set of three linear equations given by (17). The calibration constants X_i and ϕ_{xi} may be found the same way as has been described for Z and ϕ_z .

Computer Simulation

In order to prove the described method three six-port models are established and simulated with the computer. Table 1 presents the parameters of the three six-ports which consist of an ideal one as suggested by Hansson and Riblet [9], and two arbitrarily selected non-ideal ones.

Table 1.
Six-port models used in a computer simulation

Six-Port Parameters	Ideal	Non-Ideal #1	Non-Ideal #2
$Z_e^{j\phi z}$	0.0	$0.08e^{j.31\pi}$	$0.21e^{j.143\pi}$
$X_1e^{j\phi x1}$	$0.5e^{-j\frac{\pi}{2}}$	$0.58e^{-j.945\pi}$	$0.62e^{-j443\pi}$
$X_2e^{j\phi x2}$	$0.5e^{j\frac{5\pi}{6}}$	$0.70e^{j.532\pi}$	$0.52e^{j.751\pi}$
$X_3e^{j\phi x3}$	$0.5e^{j\frac{\pi}{6}}$	$0.36e^{j.11\pi}$	$0.48e^{j.155\pi}$

The three sets of offset shorts used as calibration standards are related to the following practical cases: four shorts with linearly independent offset lengths, four shorts with equal offset lengths, and a fixed short/open plus a known length of precision transmission line.

To examine the efficiency of the proposed calibration method in improving the accuracy of the calibration of a six-port reflectometer we proceeded as follows: random power reading errors of 2% and 5% were introduced and then the resultant computed values of the calibration constants were compared with their true values.

In order to compare the accuracy of the initial estimate with the final estimate of the six-port calibration constants, the following formulae has been taken as a measure of the overall error of the calibration constants.

$$M = \frac{|Z'e^{j\phi_z'}-Ze^{j\phi_z}| + \sum_{i=1}^{3} |X'_ie^{j\phi_i'}-X_ie^{j\phi_i}|}{Z + \sum_{i=1}^{3} X_i} \qquad (23)$$

where the primed symbols are related to the estimate, and nonprimed symbols to the true value of the six-port calibration constants. After the simulation of all possible combinations of the models of Table 1 and the calibration standard sets, it has been found that the calibration procedure, described here, substantially reduces the overall error of the calibration constants when random errors exist in the power detector readings. An example of computer simulation results is presented in Table 2.

Table 2
Overall errors of the calibration constants non-ideal six-port reflectometer #1

Offset Length	Δℓ	0.1π	0.2π	0.3π	0.4π	0.5π
0% Error in PMR	IEOE[%]	0.0	0.0	0.0	0.0	0.0
	FEOE[%]	0.0	0.0	0.0	0.0	0.0
2% Error in PMR	IEOE[%]	40.31	11.56	4.01	1.37	1.84
	FEOE[%]	20.71	9.21	3.60	0.87	1.77
5% Error in PMR	IEOE[%]	29.64	42.59	7.71	3.40	8.42
	FEOE[%]	11.97	39.10	4.90	3.13	8.12

PMR - Power Meter Readings
IEOE - Initial Estimate Overall Error
FEOE - Final Estimate Overall Error

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